GSQ/M-20 **1744**

MATHEMATICS

BM-362

Linear Algebra

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Define Linear Operator. **1½**

(b) Express vector (1, 2) as a linear combination of vectors (2, 0) and (1, 3). **1½**

(c) Find a if the vectors (1, –1, 3), (1, 2, –3) and (a, 0, 1) are linearly dependent. **2**

(d) Define normed vector space. **1½**

(e) Find the norm of a vector u = (2, –3, 6) and normalize this vector. **1½**

Unit I

2. (a) Prove that every n-dimensional vector space U(F) is isomorphic to F^n. **4**

(b) For $u_1 = (1, 1, -1)$, $u_2 = (4, 1, 1)$, $u_3 = (1, -1, 2)$ to be basis of R^3. Let $T : R^3 \to R^2$ be the linear transformation such that $T(u_1) = (1, 0)$, $T(u_2) = (0, 1)$, $T(u_3) = (1, 1)$. Find T. **4**

3. (a) If a finite dimensional vector space V(F) is a direct sum of its two subspaces W_1 and W_2, then $\dim V = \dim W_1 + \dim W_2$. **4**

(b) If W_1 and W_2 are subspaces of V and $\dim W_1 = 4$, $\dim W_2 = 5$ and $\dim V = 7$, then find the possible values of $\dim (W_1 \cap W_1)$. **4**

Unit II

4. (a) Prove that two finite dimensional vector spaces over the same field are isomorphic iff they have the same dimension. **4**

(b) Show that the mapping $T : R^3 \to R^2$ defined by $T(x_1, x_2, x_3) = (x_1, x_2)$ is a linear transformation and is onto but not one-to-one. **4**

5. (a) If $T : U(F) \to V(F)$ is a linear transformation, then prove that Rank T + Nullity T = dim U. **4**

(b) If $T : R^4 \to R^3$ is a linear transformation defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$ and $T(e_4) = (1, 0, 1)$. Then verify that $\rho(T) + \mu(T) = \dim R^4 = 4$. **4**

Unit III

6. (a) Prove that a linear transformation $T : U \to V$ is non-singular iff T is one-to-one. **4**

(b) If $T : R^3 \to R^3$ be a linear operator defined by $T(x, y, z) = (x + z, x - z, y)$. Show that T is invertible and find T^{-1}. **4**

7. (a) If $B = \{(1, -2, 3), (1, -1, 1), (2, -4, 7)\}$ is a basis of R^3, then find the dual basis of B. **4**

(b) Find the eigen values, eigen vectors for the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ **4**

Unit IV

8. (a) Let V be an inner product space, then prove that $|\langle u, v\rangle| \leq \|u\| \, \|v\|$ for all $u, v \in V$. **4**

(b) If x and y are vectors in an inner product space, then show that $x = y$ iff $\langle x, z\rangle = \langle y, z\rangle$ for all $z \in V$. **4**

9. (a) Prove that every finite dimensional inner product space has an orthonormal basis. **4**

(b) Let T be a linear operator on a finite dimensional inner product space V(F). It T is invertible, then show that so is T^* and $(T^*)^{-1} = (T^{-1})^*$. **4**