

Roll No.

Total Pages : 05

GSQ/M-20
MATHEMATICS
BM-363
Dynamics

1723

Time : Three Hours]

[Maximum Marks : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) A particle describes an equiangular spiral $r = ae^{2\theta}$ with constant angular velocity. Find its radial acceleration. **1**
- (b) The maximum velocity of a body moving with S.H.M. is 2 unit/sec. and its period is $\frac{1}{5}$ sec. What is its amplitude ? **1**
- (c) Define Hook's law and modulus of elasticity. **2**
- (d) Show that at an apse, the particle moves at right angle to radius vector. **2**
- (e) A particle is projected with a velocity of 49 m/sec. in a direction making an angle of 45° with the horizontal. Find (i) Time of flight (ii) Horizontal range. **1**

Unit I

2. (a) Find the expressions for tangential and normal components of acceleration of a particle moving along a plane curve. 2½
- (b) To a passenger in an open car travelling of 20 km/hr; the wind appears to come from a direction 60° to the right and from ahead at 4 km/hr. What is the true direction and velocity of the wind ? 2½
3. (a) A point moving with S.H.M. has a velocity of 6 cm/sec. when passing through the centre of its path and its period is $\frac{2\pi}{3}$ sec. Find its velocity when it has moved a distance $\frac{3}{2}$ cm from its extreme position. 2½
- (b) A light elastic string of natural length l has one end fixed at a point O and the other end attached to a particle, the weight of which in equilibrium position would extend the string to a length l_1 . Show that if the particle be dropped from rest at O, it will come to instantaneous rest at a depth $\sqrt{l_1^2 - l^2}$ below the equilibrium position. 2½

Unit II

4. (a) A mass of 10 kg falls freely a distance of 10 m from rest and is then brought to rest after penetrating through 1 m in sand. Find the average force exerted by the sand on it. 2½
- (b) A bullet weighing 81 gm and moving at the rate of 200 cm/sec. is fired into a log of wood into which it penetrates 10 cm. If the bullet moving with the same velocity were fired into a similar piece of wood 5 cm thick, with what velocity would it emerge ? Also find the force of resistance, supposing it to be uniform. 2½
5. (a) A train whose mass is 150 tons has an engine of 230 H.P. Find the greatest uniform speed that can be maintained while ascending an incline of 1 in 80, the resistance being equal to the weight of one ton. 2½
- (b) Prove that a train of weight W tons going up an incline of 1 in m will acquire a velocity $\left(\frac{P}{W} - \frac{1}{m} - \frac{R}{2240} \right)$ gt ft/sec. and kinetic energy $\frac{1}{2} W \left(\frac{P}{W} - \frac{1}{m} - \frac{R}{2240} \right)^2$ gt²ft tons after t sec. from rest, P being pull of the engine in tons and R the resistance on the level in pounds per ton. 2½

Unit III

6. (a) A particle is projected with velocity u from the lowest point and moves along the inside of a smooth vertical circle. Discuss the motion. $2\frac{1}{2}$
- (b) A particle is projected very close to the vertex of a smooth cycloid whose axis is vertical and vertex upwards and is allowed to run down the curve. Discuss its motion. $2\frac{1}{2}$
7. (a) Find the velocity and direction of projection of shot which passes in a horizontal direction over the top of a wall 64 ft. high and 192 ft. distant from the gun. $2\frac{1}{2}$
- (b) Shots fired simultaneously from the top and the bottom of a vertical cliff with elevation ' a ' and ' b ' respectively strike an object simultaneously at the same point. Show that if ' a ' is the horizontal distance of the object from the cliff, the height of cliff is a $(\tan\alpha - \tan\beta)$. $2\frac{1}{2}$

Unit IV

8. (a) A particle moving under the influence of a central force, describes a circle through the centre of the force. Prove that the force is attractive and inversely proportional to the fifth power of the distance. $2\frac{1}{2}$

- (b) A particle moves with a central acceleration

$$\left(\frac{\lambda}{(\text{distance})^3} \right). \text{ Find the path and distinguish the}$$

cases. 2½

9. (a) If v_1 and v_2 are the maximum and minimum velocities of a planet, then prove that $(1-e)v_1 + (1+e)v_2$ for an elliptic path. 2½

- (b) A smooth helix is placed with its axis vertical and a small bead slides down it under gravity. Show that it makes its first revolution from rest in time

$$2\sqrt{\frac{\pi a}{g \sin \alpha \cos \alpha}}, \text{ where } \alpha \text{ is the angle of helix. } 2½$$