

GSE/M-20**1449****MATHEMATICS****(Ordinary Differential Equations)****Paper : BM-122****Time : Three Hours]****[Maximum Marks : 26**

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) Write the solution of exact differential equation
 $M dx + N dy = 0$. 1

(b) Solve the equation $(y - px)^2 = 1 + p^2$. 1½

(c) Find the solution of equation $\frac{d^4 y}{dx^4} - a^4 x = 0$. 1½

(d) Write the condition if e^x is a particular solution of

$$\frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Qy = R. \quad 1$$

(e) Define Total differential equation. 1

SECTION-I

2. (a) Solve the equation

$$(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0. \quad 2\frac{1}{2}$$

- (b) Solve the equation

$$\left(y + \frac{y^3}{3} + \frac{x^2}{2} \right) dx + \frac{1}{4} (x + xy^2) dy = 0. \quad 2\frac{1}{2}$$

3. (a) Solve the differential equation $y = -px + x^4p^2$. $2\frac{1}{2}$

- (b) Solve the equation $\sin px \cos y = \cos px \sin y + p$, and obtain the singular solution. $2\frac{1}{2}$

SECTION-II

4. (a) Find the orthogonal trajectories of the cardioid $r = a(1 - \cos \theta)$, where a is the parameter. 2

- (b) Solve the equation $\frac{d^2y}{dx^2} + y = \sec x$. 3

5. (a) Solve the equation $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$. 3

- (b) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = \sin x \sin 2x. \quad 2$$

SECTION-III

6. (a) Solve the equation

$$x^2 \frac{d^2 y}{dx^2} - 2x(1+x) \frac{dy}{dx} + 2(1+x)y = x^3. \quad 2\frac{1}{2}$$

- (b) Solve the equation by removing the first derivative :

$$\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = (\sec x)e^x. \quad 2\frac{1}{2}$$

7. (a) Solve the equation by variation of parameters :

$$\frac{d^2 y}{dx^2} - y = \frac{2}{1+e^x}. \quad 2\frac{1}{2}$$

- (b) Solve the equation by changing the independent variable :

$$\frac{d^2 y}{dx^2} - \cot x \frac{dy}{dx} - y \sin^2 x = 0. \quad 2\frac{1}{2}$$

SECTION-IV

8. (a) Solve the simultaneous equations

$$t \frac{dx}{dt} + y = 0$$

$$t \frac{dy}{dt} + x = 0$$

$$\text{given that } x(1) = 1, y(-1) = 0. \quad 3$$

(b) Solve the equation

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (y+x)^2}. \quad 2$$

9. (a) Solve the total differential equation

$$2yz \, dx + zx \, dy - xy(1+z) \, dz = 0. \quad 2\frac{1}{2}$$

(b) Solve the equations :

$$\frac{dx}{dt} + 5x + y = e^t.$$

$$\frac{dy}{dt} - x + 3y = e^{2t}. \quad 2\frac{1}{2}$$
