

GSE/M-20**1473****MATHEMATICS****(Ordinary Differential Equations)****Paper : BM-122**

Time : Three Hours]

[Maximum Marks : 40

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) Define integrating factor of a differential equation. 1

(b) Define Total differential equation. 1

(c) Find the value of p from the differential equation

$$xyp^2 + p(3x^2 - 2y^2) - 6xy = 0. \quad 2$$

(d) Solve the differential equation $\frac{d^2y}{dx^2} + y = x$. 2

(e) Show that the equation

$$(x - 3y - z) dx + (2y - 3x) dy + (z - x) dz = 0$$

is exact. 2

SECTION-I

2. (a) Solve the differential equation

$$(1 + e^{x/y}) dx + e^{x/y} \left(1 - \frac{x}{y} \right) dy = 0. \quad 4$$

- (b) Solve the differential equation

$$(x^2 + y^2 + x) dx + xy dy = 0. \quad 4$$

3. (a) Solve the differential equation

$$p^3 - p(x^2 + xy + y^2) + xy(x + y) = 0. \quad 4$$

- (b) Find the complete primitive and singular solution of

$$xp^2 - 2yp + 4x = 0. \quad 4$$

SECTION-II

4. (a) Find the orthogonal trajectories of the cardioid

$$r = a(1 - \cos \theta), \text{ where } a \text{ is the parameter.} \quad 4$$

- (b) Solve the differential equation $\frac{d^2 y}{dx^2} + y = \sec x.$ 4

5. (a) Solve the differential equation

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x. \quad 4$$

- (b) Solve the differential equation $\frac{d^4 y}{dx^4} - y = x \sin x + e^x.$

4

SECTION-III

6. (a) Solve the differential equation

$$(x^2 \theta^2 - 3x\theta + 5)y = \sin(\log x). \quad 4$$

- (b) Solve the differential equation by removing the first derivative :

$$\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = (\sec x) e^x. \quad 4$$

7. (a) Solve the differential equation by changing the independent variable :

$$\cos x \frac{d^2 y}{dx^2} + \sin x \frac{dy}{dx} - 2y \cos^3 x = 2 \cos^5 x. \quad 4$$

- (b) Apply the method of variation of parameters to solve

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x \log x. \quad 4$$

SECTION-IV

8. (a) Solve the simultaneous equations :

$$\frac{d^2 x}{dt^2} - 3x - 4y = 0.$$

$$\frac{d^2 y}{dt^2} + x + y = 0. \quad 4$$

- (b) Solve the simultaneous equations :

$$\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z + \frac{1}{z}}. \quad 4$$

9. (a) Solve the equation $(yz + z^2) dx - xz dy + xy dz = 0$. 4
- (b) Solve the differential equation
 $(y^2 + yz) dx + (xz + z^2) dy + (y^2 - xy) dz = 0$. 4
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