

Roll No.

Total Pages : 05

GSQ/M-20
MATHEMATICS
BM-363
Dynamics

1745

Time : Three Hours]

[Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

Compulsory Question

1. (a) A particle moves in a plane, its velocities parallel to the axes of x and y being $u + ey$ and $v + ex$ respectively. Show that it moves in a conic section. 1½
- (b) A particle moving with S.H.M. of period 12 seconds travels 10 cm from the position of rest in 2 seconds. Find the amplitude, the maximum velocity and the velocity at the end of 2 seconds. 2
- (c) A body of mass 25 gms is acted upon by a constant force. It acquires a velocity of 2 cm/sec. in 5 seconds from rest. Find, how large is the force acting. 2

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- (d) Two balls are projected from the same point in directions inclined at 60° and 30° to the horizontal. If they attain the same height, what is the ratio of their velocities of projection ? $1\frac{1}{2}$
- (e) Write Differential Equation of central orbit in polar form. 1

Unit I

2. (a) Find the expressions for tangential and normal components of acceleration of particle moving along a plane curve. 4
- (b) A passenger travelling in a train with velocity 90 km/hr on a straight level track observes that another train which is 180 m long and moving constant takes 4 seconds to pass by. What is the velocity of passing train ? 4
3. (a) A particle is describing S.H.M. of period T along a straight line. If v be its speed when at a distance x from the mean and a is the amplitude, show that $v^2 T^2 = 4\pi^2 (a^2 - x^2)$. 4
- (b) An elastic string of natural length l and modulus of elasticity λ has one end fixed at a point O on a smooth horizontal table. A particle of mass m is attached to the other end A pulled to the position B , where $AB = a$ and then let go. Discuss the motion. 4

Unit II

4. (a) A mass of 10 kg falls freely a distance of 10 m from rest and is then brought to rest after penetrating through 1 m in sand. Find the average force exerted by the sand on it. 4
- (b) Two masses m_1 and m_2 ($m_1 > m_2$) are suspended by a light inextensible and flexible string which passes over a smooth fixed and light pulley. To find the motion of the system, the tension in the string and pressure on pulley. 4
5. (a) Show that in any displacement of a particle, the change in the K.E. is equal to work done by the impressed forces acting on the particle. 4
- (b) A train of mass M lbs is ascending a smooth incline of 1 in n and when the velocity of train is v ft/sec, its acceleration is f ft/sec². Prove that the effective horse power of engine is $\frac{Mv(nf + g)}{550 ng}$. 4

Unit III

6. (a) A small bead is projected with any velocity along a smooth circular wire under the action of force

varying inversely as the fifth power of distance from a centre of force situated on the circumference.

Prove that pressure on wire is constant. 4

- (b) A particle is projected with velocity ' u ' from the lowest point and moves along the inside of a smooth vertical circle. Discuss the motion. 4

7. (a) A bomber is flying at a constant horizontal velocity of 210 km/hr at a height of 1000 metres above the ground towards the point directly above the target. At what angle of sight should a bomb be dropped so as to hit the target ? 4

- (b) If R is the maximum range on an inclined plane through the point of projection of a particle and T the corresponding time of flight, show that

$$R = \frac{1}{2}gT^2. \quad 4$$

Unit IV

8. (a) Prove that central orbit is always a plane curve. 4
(b) A particle describes the equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force to the pole. Find the law of force. 4

9. (a) If a planet were suddenly stopped in its orbit when at a distance ' a ' from the sun, show that it would fall in the sun in time $\frac{\sqrt{2}\pi a^{3/2}}{4\sqrt{\mu}}$ which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 4
- (b) A particle moves on a smooth sphere under no force except the pressure of the surface. Show that its path is given by the equation $\cot \theta = \cot \beta \cos \phi$, where θ and ϕ are its angular co-ordinates. 4